Accurate Modeling of Antennas Using Variable-Fidelity EM Simulations and Co-Kriging

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Abstract—**We present an accurate and low-cost modeling of antenna structures using variable-fidelity electromagnetic (EM) simulations. Our approach exploits sparsely sampled high-fidelity (accurate) EM data as well as densely sampled coarse-discretization (low-fidelity) EM simulations that are accommodated into one model using co-kriging technique. By using coarse-discretization simulations, the computational cost of creating the antenna model is greatly reduced compared to conventional approach, where high-fidelity simulations are directly used to set up the model. To our knowledge, this is the first application of co-kriging to antenna modeling. Numerical verification and comparisons with kriging interpolation are given.**

Keywords-Antenna modeling; electromagnetic (EM) simulation; kriging; co-kriging; computer-aided design (CAD).

I. INTRODUCTION

Reliable evaluation of antenna structures can be obtained through electromagnetic (EM) simulation. High-fidelity simulation is CPU intensive, which is a bottleneck for EM-based design tasks such as parametric optimization, statistical analysis, or yield-driven design. Thus, accurate and computationally cheap models of antennas (so-called surrogates) are indispensable.

Cheap antenna models can be obtained using approximation techniques such as polynomial regression [1], radial basis functions [2], kriging [2], [3], support vector regression [4]-[6], artificial neural networks [7]-[10], fuzzy systems [11], or multidimensional Cauchy approximation [12]. However, for good accuracy, these techniques require a large number of training points, particularly if the number of design variables is large.

Here, we consider antenna models constructed using both high- and low-fidelity EM simulations. Simulation of coarsely-discretized antenna structure may not be accurate; however, it is much faster than the high-fidelity one. As we demonstrate, such low-fidelity data can be combined with sparsely sampled high-fidelity simulations using co-kriging [13]. The resulting antenna model is as accurate as the conventional approximation surrogate using much larger number of training data points. The proposed modelling technique is demonstrated using two examples: a ultrawideband planar dipole antenna and a rectangular dielectric resonator antenna. Comparison with conventional kriging interpolation is also given.

II. ANTENNA MODELING USING CO-KRIGING

A. Antenna Models

We consider two types of antenna models. Let $\boldsymbol{R}_f(\boldsymbol{x})$ denote an EM-simulated high-fidelity model, which is an accurate representation of the antenna structure. $\boldsymbol{R}_f$ is expensive to evaluate (typical simulation time measured in hours). Here, $\boldsymbol{x}$ is a vector of designable (e.g., geometry) parameters. The components of $\boldsymbol{R}_f$ may represent, e.g., the antenna reflection coefficient $|S_{11}|$ over the frequency band of interest. We also consider an auxiliary (low-fidelity) model $\boldsymbol{R}_c$ which may be evaluated using the same EM solver, however, with coarser discretization. The low-fidelity model $\boldsymbol{R}_c$ is much faster than $\boldsymbol{R}_f$ but not as accurate. Therefore, it cannot be normally directly used instead of the high-fidelity model to perform tasks such as design optimization. In this paper, we combine the low- and high-fidelity simulations to create the surrogate model that is almost as accurate as $\boldsymbol{R}_f$ but requires much smaller number of high-fidelity training points than the approximation model created using only $\boldsymbol{R}_f$ samples.

B. Kriging Interpolation

Kriging is a popular technique to interpolate deterministic noise-free data [2], [14]. These Gaussian Process based surrogate models are compact and cheap to evaluate. Here, we use kriging as a benchmark technique for comparison with the co-kriging of Section II.*C*. Let $X_{B.KR} = \{\boldsymbol{x}_{KR}^1, \boldsymbol{x}_{KR}^2, \ldots, \boldsymbol{x}_{KR}^{NKR}\} \subset X_R$ be the base (training) set and $\boldsymbol{R}_f(X_{B.KR})$ the associated fine model responses. Then, the kriging interpolant, also known as the Best Linear Unbiased Predictor (BLUP), is derived as,

$$\boldsymbol{R}_{s.KR}(\boldsymbol{x}) = M\alpha + r(\boldsymbol{x}) \cdot \Psi^{-1} \cdot (\boldsymbol{R}_f(X_{B.KR}) - F\alpha) \tag{1}$$

where M and F are Vandermonde matrices of the test point $\boldsymbol{x}$ and the base set $X_{B.KR}$, respectively. The coefficient vector α is determined by Generalized Least Squares (GLS). $r(\boldsymbol{x})$ is an $1\times N_{KR}$ vector of correlations between the point $\boldsymbol{x}$ and the base set $X_{B.KR}$, where the entries are $r_i(\boldsymbol{x}) = \psi(\boldsymbol{x},\boldsymbol{x}_{KR}^i)$, and Ψ is a

$N_{KR} \times N_{KR}$ correlation matrix, with the entries given by $\Psi_{i,j} = \psi(\boldsymbol{x}_{KR}^i, \boldsymbol{x}_{KR}^j)$. In this work, the exponential correlation function is used, i.e., $\psi(\boldsymbol{x},\boldsymbol{y}) = \exp(\sum_{k=1,\ldots,n} -\theta_k |x^k - y^k|)$, where the parameters $\theta_1, \ldots, \theta_n$ are identified by Maximum Likelihood Estimation (MLE). The regression function is chosen constant, $F = [1 \ \ldots \ 1]^T$ and $M = (1)$.

C. Co-Kriging Modeling

Co-kriging [13] is a type of kriging where the $\boldsymbol{R}_f$ and $\boldsymbol{R}_c$ model data are combined to enhance the prediction accuracy. Co-kriging is a two-steps process: first a kriging model $\boldsymbol{R}_{s.KRc}$ of the coarse data $(X_{B.KRc}, \boldsymbol{R}_c(X_{B.KRc}))$ is constructed and on the residuals of the fine data $(X_{B.KRf}, \boldsymbol{R}_d)$ a second kriging model $\boldsymbol{R}_{s.KRd}$ is applied, where $\boldsymbol{R}_d = \boldsymbol{R}_f(X_{B.KRf}) - \rho \cdot \boldsymbol{R}_c(X_{B.KRf})$. The parameter ρ is included in the MLE. Note that if the response values $\boldsymbol{R}_c(X_{B.KRf})$ are not available, they can be approximated by using the first kriging model $\boldsymbol{R}_{s.KRc}$, namely, $\boldsymbol{R}_c(X_{B.KRf}) \approx \boldsymbol{R}_{s.KRc}(X_{B.KRf})$. The resulting co-kriging interpolant is defined as

$$\boldsymbol{R}_{s.CO}(\boldsymbol{x}) = M\alpha + r(\boldsymbol{x}) \cdot \Psi^{-1} \cdot (\boldsymbol{R}_d - F\alpha) \tag{2}$$

where the block matrices M, F, $r(\boldsymbol{x})$ and Ψ can be written in function of the two separate kriging models $\boldsymbol{R}_{s.KRc}$ and $\boldsymbol{R}_{s.KRd}$:

$$r(\boldsymbol{x}) = [\rho \cdot \sigma_c^2 \cdot r_c(\boldsymbol{x}), \rho^2 \cdot \sigma_c^2 \cdot r_c(\boldsymbol{x}, X_{B.KR_f}) + \sigma_d^2 \cdot r_d(\boldsymbol{x})]$$

$$\Psi = \begin{bmatrix} \sigma_c^2 \Psi_c & \rho \cdot \sigma_c^2 \cdot \Psi_c(X_{B.KR_c}, X_{B.KR_f}) \\ 0 & \rho^2 \cdot \sigma_c^2 \cdot \Psi_c(X_{B.KR_f}, X_{B.KR_f}) + \sigma_d^2 \cdot \Psi_d \end{bmatrix} \tag{3}$$

$$F = \begin{bmatrix} F_c & 0 \\ \rho \cdot F_d & F_d \end{bmatrix}, \quad M = [\rho \cdot M_c \ \ M_d]$$

where $(F_c, \sigma_c, \Psi_c, M_c)$ and $(F_d, \sigma_d, \Psi_d, M_d)$ are matrices obtained from the kriging models $\boldsymbol{R}_{s.KRc}$ and $\boldsymbol{R}_{s.KRd}$, respectively (see Section II.*B*). In particular, σ_c^2 and σ_d^2 are process variances, while $\Psi_c(\cdot,\cdot)$ and $\Psi_d(\cdot,\cdot)$ denote correlation matrices of two datasets with the optimized $\theta_1, \ldots, \theta_n$ parameters and correlation function of the kriging models $\boldsymbol{R}_{s.KRc}$ and $\boldsymbol{R}_{s.KRd}$, respectively.

III. Verification Examples

A. UWB Planar Dipole Antenna

Consider the planar dipole antenna [15] (Fig. 1). The design variables are $\boldsymbol{x} = [l_0 \ w_0 \ a_0 \ l_p \ w_p \ s_0]^T$. The high-fidelity model $\boldsymbol{R}_f$ (~10 mln mesh cells, evaluation time 44 minutes) is simulated using the CST MWS transient solver [16]. The low-fidelity model $\boldsymbol{R}_c$ is also evaluated in CST (~100,000 mesh cells, evaluation time 43 seconds). The antenna models are set up in the region with the center at $\boldsymbol{x}^0 = [19 \ 13 \ 0.5 \ 13 \ 6 \ 1]^T$ and size $\boldsymbol{\delta} = [1 \ 1 \ 0.2 \ 1 \ 1 \ 0.2]^T$. The kriging and co-kriging models $(\boldsymbol{R}_{s.KR}, \boldsymbol{R}_{s.CO})$ are constructed using various numbers of training points (from $N_{KR} = 20$ to $N_{KR} = 400$). Co-kriging models are configured using 400 $\boldsymbol{R}_c$ samples (the CPU cost of which corresponds to around 6 evaluations of $\boldsymbol{R}_f$). The quality of the surrogate is assessed using a relative error measure $||\boldsymbol{R}_f(\boldsymbol{x}) - \boldsymbol{R}_s(\boldsymbol{x})||/||\boldsymbol{R}_f(\boldsymbol{x})||$ expressed in percent. The error is averaged over 50 test designs.

The modeling errors are given in Table I (see also Fig. 2). Note that the co-kriging model accuracy obtained with 20 (50) $\boldsymbol{R}_f$ samples is as good as that of the kriging model obtained for 100 (200) samples, which proves that co-kriging and the use of coarse-discretization EM data allows us to greatly reduce the CPU cost of creating accurate antenna model compared to conventional method using solely $\boldsymbol{R}_f$ information.

B. Rectangular Dielectric Resonator Antenna

Consider the rectangular suspended DRA [17] (Fig. 3). The design variables are $\boldsymbol{x} = [\varepsilon_1 \ h_1 \ h_2 \ s_1 \ w_1]^T$. Other parameters are fixed. The high- and low-fidelity models are evaluated in CST [16] with the following evaluation times: $\boldsymbol{R}_f$ 11 minutes, and $\boldsymbol{R}_c$ 20 sec. The antenna models are set up in the region with the center at $\boldsymbol{x}^0 = [10 \ 8.5 \ 0.5 \ 3 \ 10]^T$ and size $\boldsymbol{\delta} = [1 \ 1 \ 0.5 \ 1 \ 2]^T$ mm. Similarly as for the previous example, co-kriging allows to substantially reduce the computational cost of creating the accurate antenna model when compared to approximation of the high-fidelity model data only (cf. Table II and Fig. 4).

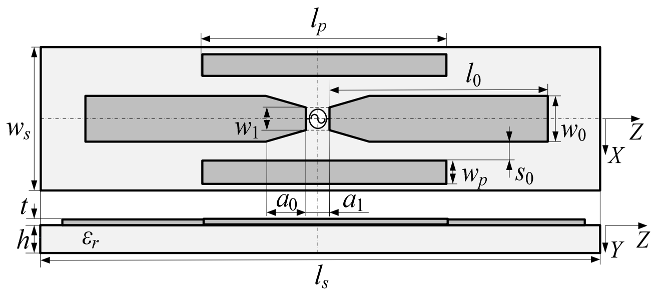

Fig. 1. Dipole antenna geometry [15]: top and side views. The dash-dot lines show the magnetic (YOZ) and the electric (XOY) symmetry walls. The 50 ohm source impedance is not shown at the figure.

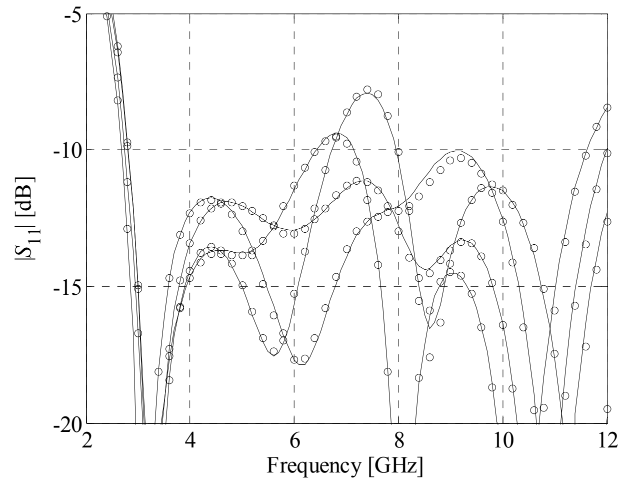

Figure 2. UWB dipole: responses of $\boldsymbol{R}_f$ (—) and co-kriging surrogate model (o) at selected test points. Co-kriging model created using 50 evaluations of $\boldsymbol{R}_f$ and 400 evaluations of $\boldsymbol{R}_c$.

IV. CONCLUSION

We presented an antenna modeling methodology using co-kriging. We demonstrate that by combining the low- and high-fidelity EM simulations, it is possible to create an accurate model of an antenna structure while using limited number of high-fidelity data points.

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TABLE I. UWB DIPOLE ANTENNA: MODELING RESULTS

Model	Average Modeling Error [%]				
	$N_{KR} = 20$	$N_{KR} = 50$	$N_{KR} = 100$	$N_{KR} = 200$	$N_{KR} = 400$
$\boldsymbol{R}_{s.KR}$	17.5	5.6	4.3	2.8	2.0
$\boldsymbol{R}_{s.CO}$	4.2	2.6	2.4	2.0	1.9

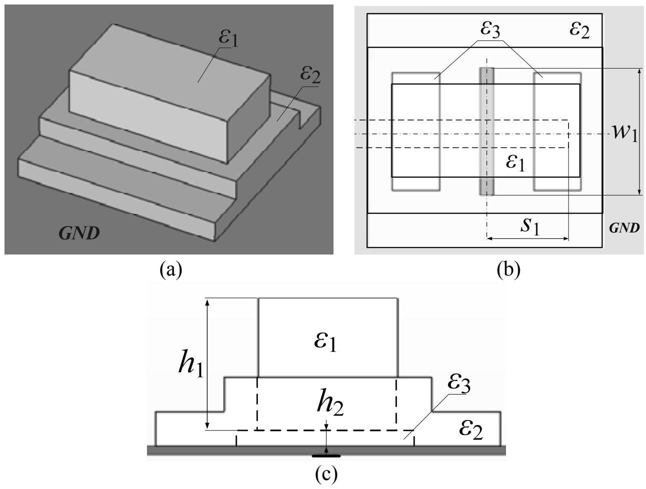

Fig. 1. DRA geometry [17]: (a) 3D view, (b) top view, and (c) side view.

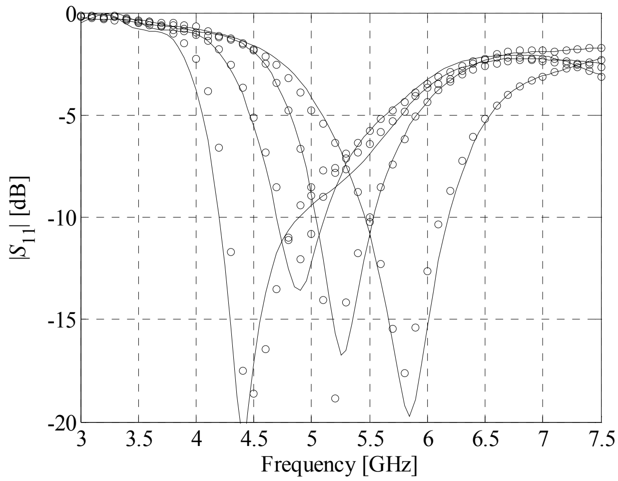

Figure 2. Rectangular DRA: responses of $\boldsymbol{R}_f$ (—) and co-kriging surrogate (o) at selected test points. Co-kriging model created using 50 evaluations of $\boldsymbol{R}_f$ and 400 evaluations of $\boldsymbol{R}_c$.

TABLE II. RECTANGULAR DRA: MODELING RESULTS

Model	Average Modeling Error [%]				
	$N_{KR} = 20$	$N_{KR} = 50$	$N_{KR} = 100$	$N_{KR} = 200$	$N_{KR} = 400$
$\boldsymbol{R}_{s.KR}$	12.1	8.8	6.9	5.2	3.6
$\boldsymbol{R}_{s.CO}$	6.7	5.4	5.0	4.1	3.5